B.Sc. part-I (S), Set Theory 06-07-2020

De Morgan's Laws in another form

Q. 1. Prove that
$$A-(B\cap C)=(A-B)\cup(A-C)$$

~~Prove that~~
Proof: Let x be any arbitrary element of $A-(B\cap C)$, then

$$x\in A-(B\cap C)\Rightarrow x\in A \wedge x\notin (B\cap C)$$
$$\Rightarrow x\in A \wedge (x\notin B \vee x\notin C)$$
$$\Rightarrow (x\in A\wedge x\notin B)\vee(x\in A\wedge x\notin C)$$
$$\Rightarrow x\in (A-B)\vee x\in(A-C)$$
$$\Rightarrow x\in (A-B)\cup(A-C)$$

$$\therefore A-(B\cap C)\subseteq (A-B)\cup(A-C) \quad\text{—— (1)}$$

Again, Let y be any arbitrary element of $(A-B)\cup(A-C)$

Then,
$$y\in(A-B)\cup(A-C)\Rightarrow y\in(A-B)\vee y\in(A-C)$$
$$\Rightarrow (y\in A\wedge y\notin B)\vee(y\in A\wedge y\notin C)$$
$$\Rightarrow (y\in A\wedge y\notin B)\vee(y\in A\wedge y\notin C)$$
$$\Rightarrow y\in A\wedge(y\notin B\vee y\notin C)$$
$$\Rightarrow y\in A\wedge y\notin(B\cap C)$$
$$\Rightarrow y\in A-(B\cap C)$$

$$\therefore (A-B)\cup(A-C)\subseteq A-(B\cap C) \quad\text{—— (2)}$$

From (1) and (2), we get
$$A-(B\cap C)=(A-B)\cup(A-C)$$
Proved.

Q. 2. If A, B and C are three sets, then
$$A\Delta(B\Delta C)=(A\Delta B)\Delta C.$$

Proof. Let $x\in A\Delta(B\Delta C)\Leftrightarrow x\in[A-(B\Delta C)]\cup[(B\Delta C)-A]$

$\Leftrightarrow x\in[A-(B\Delta C)]\vee x\in[(B\Delta C)-A]$

$\Leftrightarrow [x\in A\wedge x\notin(B\Delta C)]\vee[x\in(B\Delta C)\wedge x\notin A]$

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Rest part of Q. 2.

$\Leftrightarrow [x \in A \wedge x \in (B \cap C) \vee x \in A \wedge x \notin (B \cup C)]$

$\vee [(x \in B - C \wedge x \notin A) \vee (x \in C - B \wedge x \notin A)]$

$\Leftrightarrow [(x \in A \wedge x \in B \wedge x \in C) \vee (x \in A \wedge x \notin B \wedge x \notin C)]$

$\vee [x \in B \wedge x \notin C \wedge x \notin A] \vee (x \in C \wedge x \notin B \wedge x \notin A)]$

$\Leftrightarrow (x \in A \cap B \wedge x \in C) \vee [x \in A - B \wedge x \notin C)$

$\vee (x \in B - A \wedge x \notin C) \vee (x \notin A \cup B \wedge x \in C)$

$\Leftrightarrow [x \in (A-B) \vee x \in (B-A)] \wedge x \notin C \vee$

$[x \in (A \cap B) \vee x \notin (A \cup B)] \wedge x \in C$

$\Leftrightarrow [x \in (A-B) \cup (B-A) \wedge x \notin C] \vee [x \notin (A \Delta B) \wedge x \in C]$

$\Leftrightarrow [x \in (A \Delta B) \wedge x \notin C] \vee [x \in C \wedge x \notin (A \Delta B)]$

$\Leftrightarrow [x \in (A \Delta B) - C] \vee x \in [C - (A \Delta B)]$

$\Leftrightarrow [x \in [(A \Delta B) - C] \cup [C - (A \Delta B)]$

$\Leftrightarrow x \in (A \Delta B) \Delta C$

Thus, $A \Delta (B \Delta C) = (A \Delta B) \Delta C$

Proved.